ATTENUATION OF WAVES PROPAGATING ALONG A SURFACE OF A NONHOMOGENEOUS SEMISPACE WITH A WAVEGUIDE

PMM Vol. 31, No. 5, 1967, pp. 914-915

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(Received May 29, 1967)

We study the surface wave solutions in the problem on natural high frequency oscillations of a layer-wise nonhomogeneous semispace whose refraction index has, at first, a maximum value which gradually decreases to a minimum and then, beginning from a certain depth, assumes finally a constant value. A boundary condition of the third kind is given on the surface, and the condition of emission of radiation, at infinity. This leads to quasi-intersection of dispersion curves and to the displacement of eigenvalues into a complex plane. The latter results in attenuation of waves propagating along the boundary, with the rate of attenuation different for various types of waves. During our investigation we use the method of joining the solutions of known asymptotics over the intervals, in which the index of refraction behaves monotonously.

Let us consider the propagation of waves in a medium nonhomogeneous in the vertical direction, contained in a semispace $z \geq 0$ and defined by the wave equation

$$\Delta U = \frac{1}{c^2(z)} U_{tt} \tag{1}$$

Here the velocity of the volume waves $c(z)$ is a sufficiently smooth function of the z-coordinate, it has a minimum at $z = e_1$, a maximum at $z = e_2$ $(0 < e_1 < e_2)$ and is constant when $z > e_3 > e_2$ with $c'(z) \neq 0$ on the intervals of monotonous behavior of $c(z)$ (Fig. 1). We shall seek the solutions of (1) in the form of a plane wave with its phase velocity equal to σ,

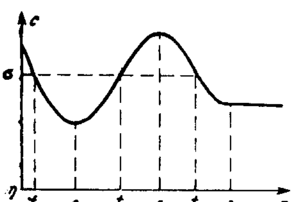

Fig. 1

$$U(x, y, z, t) = e^{ik(x-t\sigma)} V(z, k, \sigma) \tag{2}$$

Let us introduce the notation

$$n(z) = 1 / c(z), \qquad m^2(z, \sigma) = 1 - n^2(z)\sigma^2$$

When $z \geq 0$, we have for V

$$V'' - k^2 m^2(z, \sigma) V = 0 \qquad \left(' = \frac{d}{dz}\right) \tag{3}$$

We shall use the condition of third kind

$$V' + k\alpha V = 0 \qquad (0 < \alpha < 1) \tag{4}$$

as the boundary condition at $z = 0$.

Physically, such a condition is satisfied by the waves on a solid-liquid interface (see the candidate dissertation of V. Iu. Zavadskii, Acoustic Institute, M., 1965). In analogous quantum-mechnical problems, this condition is equivalent to an addition of a coupled state.

In the present paper, we study the asymptotic behavior (for $k \to \infty$) of some roots on the σ-plane of the characteristic equation of a problem obtained from (3) and (4) by addition of a condition prevailing when $z \to \infty$ (we easily see that a discrete spectrum occupies the interval $0 < \sigma^2 < c^2(e_3)$, while a continuous spectrum lies in the interval $\sigma^2 > c^2(e_3)$). A detailed discussion is given in another paper, here we consider only a region of phase velocities $\sigma \approx \sigma_0$ where $\sigma_0 = c(0)\sqrt{1-\alpha^2}$ is the velocity of a surface wave in a homogeneous semispace with $c(z) \equiv c(0)$ (such a wave analogous to a Rayleigh wave in an elastic medium, can be generated by the condition (4)). Moreover, let

$$\max\{c(e_1),\ c(e_3)\} < \sigma_* < \min\{c(0),\ c(e_2)\}$$

We shall show that in the present case, resonance effects (quasi-transverse) and a shift of the eigenvalues into a complex space can be observed.

Let I_j be an interval in which $c(z)$ behaves monotonously, let t_j be a stagnation point on I_j ($j = 1, 2, 3$) and let the latter terminate at the right-hand side point e_j.

We shall introduce the following notation

$$\varphi_j = \left(\frac{3}{2}\int_{t_j(\sigma)}^{z} m(\zeta, \sigma)\, d\zeta\right)^{2/3}, \quad z \in I_j; \quad \varphi_j > 0 \text{ when } c(z) > \sigma$$

$$W_j = \begin{pmatrix} w_{j1} & w_{j2} \\ k^{-1}w_{j1}' & k^{-1}w_{j2}' \end{pmatrix} \qquad w_{j\nu} = \frac{1}{\sqrt{\varphi_j'}} A_\nu (k^{2/3}\varphi_j)$$

where $A_\nu(\tau)$ is the Airy function and $\nu = 1, 2$.

Let us replace Eq. (3) with

$$\mathbf{Y}' = kB\mathbf{Y}, \qquad B = \begin{pmatrix} 0 & 1 \\ m^2 & 0 \end{pmatrix}, \qquad \mathbf{Y} = \begin{pmatrix} V \\ k^{-1}V' \end{pmatrix} \tag{5}$$

We retain the fundamental matrix Y of (5) on the semiaxis $z \geq 0$ by joining the fundamental matrices Y_j defined and possessing the following asymptotics on I_j

$$Y_j = [E + O(k^{-1})]\, W_j, \qquad E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

We note that since the exact solutions are joined (instead of the more usual principal terms of the asymptotic), we can take into account all the errors present in the asymptotic formulas. A solution of (5) satisfying the condition of radiation at $\sigma > c(e_3)$, $z \to +\infty$ is obtained from a complex linear combination of the columns of Y. Boundary condition at $z = 0$ gives a dispersion equation for $\sigma(k)$ of the form

$$\Phi + i\Psi = 0 \tag{7}$$

where

$$\Phi = 2R(\cos kf_2 + O_1) + Se^{-2kf_1}(\sin kf_2 + O_1)$$

$$\Psi = -e^{-2kf_3}[R(\sin kf_2 + O_1) - {}^1\!/_2\, Se^{-2kf_1}(\cos kf_2 + O_1)]$$

$$R = \alpha - m(0, \sigma) + O_1, \qquad S = \alpha + m(0, \sigma) + O_1, \qquad O_1 = O(k^{-1})$$

$$f_1 = \int_0^{t_1} m\, dz, \qquad f_2 = \int_{t_1}^{t_2} \sqrt{n^2\sigma^2 - 1}\, dz, \qquad f_3 = \int_{t_2}^{t_3} m\, dz, \quad f_j > 0$$

Neglecting Ψ in (7) we obtain a family of real dispersion curves, which quasi-intersect as in [1 and 2]. Presence of Ψ results in a shift of the roots of the dispersion equation into the complex plane and $\epsilon \equiv \operatorname{Im}\sigma < 0$, the latter condition causes the attenuation of waves in the x-direction. Different parts of dispersion curves have different attenuation rates. For the waves determined by the character of the boundary condition, we have

$$\varepsilon(k) = -(a + O_1)\exp(-2kf_1 - 2kf_3), \qquad a > 0 \tag{8}$$

while in the case of waves generated by the waveguide, i.e. by the interval (t_1, t_2), we have

$$\varepsilon(k) = -(b + O_1)\exp(-2kf_3), \qquad b > 0 \tag{9}$$

This method makes possible a generalization to the case of a multi-extremal function $c(z)$. The case of decaying waves when $c(z)$ is monotonous, was studied previously by V. Iu. Zavadskii. Under the quantum-mechanical treatment, such solutions describe quasi-stationary states [3].

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Translated by L. K.

MOTION OF GAS BEHIND AN EXPANDING DETONATION WAVE IN SPACE WITH A CUT-OUT CONE

PMM Vol. 31, No. 5, 1967, pp. 916-928
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(Received February 16, 1967)

Motion of gas behind a detonation wave expanding from the point of ignition O (coordinate origin) in a space filled with an explosive and with a cut-out hollow cone (axis of the cone: $x = 0$, $y = 0$, $z \leq 0$), possesses a cylindrical symmetry and is self-similar. Consequently, all gas-dynamic magnitudes are functions of two independent variables $\xi = r/t$, $\eta = z/t$, $r = \sqrt{x^2 + y^2}$ (here t denotes time). These functions satisfy the gas-dynamic equations with the corresponding boundary conditions, written in terms of these variables. Numerical methods of solution of partial differential equations (in two independent variables ξ and η) must however be used to obtain the above magnitudes.

S. K. Godunov assumed that a region exists on the $\xi\eta$-plane, where the flow coincides with the corresponding spherically symmetric flow obtained by Zel'dovich [1]. The latter flow occurs when a detonation wave expands from the origin O, the whole space being filled with an explosive. The motion of the gas is, in this case, spherically symmetric and self-similar, and determination of gas-dynamic functions reduces to the integration of a system of ordinary differential equations with the corresponding boundary conditions.